

Assessing Proxies for Market Prices of Thinly Traded Assets with Scheduled Cash Flows

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Abstract

Pseudo-market prices of infrequently traded assets with scheduled cash flows - commercial real estate appraisals and matrix prices of commercial mortgage backed securities - are compared against a VAR model to assess the extent to which these widely-used proxies are grounded in economic fundamentals. Property appraisals fail to fully incorporate the economic fundamentals underlying commercial real estate transactions. During the financial crisis, CMBS matrix prices captured underlying economic fundamentals and exhibited little pricing bias. However, matrix prices no longer exhibited such economic discipline after the financial crisis. Incorporating VAR forecasts considerably improves the predictive ability of appraisals and matrix prices.

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1 Introduction

Asset prices are typically thought of as being determined by arm's length transactions in an active market with frequent trades. Many large asset markets, however, are plagued by illiquidity in which few if any trades occur. For example, commercial properties and leveraged loans, important asset classes in the portfolios of pension funds, banks, life insurance companies, and other institutional investors, trade infrequently and irregularly through time.

Participants in these illiquid markets are often required to approximate the market value of their financial positions on a regular basis. This need arises because of regulatory requirements, risk management practices or financial accounting reporting. As a result, various means have been devised to approximate the market prices of assets that are not actively traded. For example, appraisals are routinely used in commercial real estate while matrix prices¹ are often relied upon in fixed income markets.

Notwithstanding their wide use in practice, there is limited previous research into the properties of these pseudo-market prices. Commercial property appraisals have been shown, on average, to deviate substantially and systematically from subsequent sales prices.² Likewise, matrix prices used in corporate bond markets can differ significantly from transaction prices at which these bonds trade.³

However, little is known *why* these proxies differ from transaction prices. Guided by

¹Traditionally, matrix pricing refers to the practice of estimating the market price of a nontraded bond using the market prices of traded bonds with similar credit quality, maturity, and coupon rates. More recently, matrix prices refer to algorithmically determined prices of nontraded securities. The models used by Bloomberg, Interactive Data Corporation (IDC) and other pricing services to arrive at these estimates are proprietary.

²Appraisals appear to lag sales prices, falling below sales prices in hot markets and remaining above sales prices in cold markets. See, for example, Cannon and Cole (2011)

³See Ferrell, Roper and Shu (2018).

economic theory, this paper investigates the circumstances under which pseudo-market prices of infrequently traded assets differ from transaction prices. In particular, we assess the extent to which pseudo-market prices differ from transaction prices because pseudo-market prices do not reflect underlying economic fundamentals prevailing in a market.

To do so requires that we estimate prices that reflect underlying economic fundamentals when assets trade infrequently. Our methodology is predicated on the fact that even if an asset trades infrequently, in many instances extensive information about the asset's cash flows are available. For example, debt service payments on many fixed income securities are often known even if the asset rarely if ever trades. Relying on this cash flow information allows us to estimate the prices of these infrequently traded assets that pseudo-market prices can be compared to and, as we document, often more accurately correspond to actual transaction prices than pseudo-market prices.

Our starting point is the observation that the price of an asset reflects the present value of the future cash flows that investors owning the asset are expected to receive. For example, the simple Gordon growth model gives that the current price of an asset, P , can be expressed as

$$P = \frac{CF}{r-g}$$

where CF is the cash flow expected to be generated by the asset over the next period, r is a risk-adjusted discount rate, and g is the assumed constant growth rate in the asset's cash flows. According to the price formation process embodied in the simple Gordon growth model, markets assess the risk attributes and growth potential of an asset's expected cash flows to determine the asset's so-called cash flow yield, $\frac{CF}{P}$. Even if the asset does not actively trade, if we know the market's assessment of its cash flow yield, or at least are able to accurately forecast it, given its cash flow, we can infer the asset's price.

We rely on the dynamic version of the Gordon growth model (Campbell and Shiller (1988)) in implementing our methodology. Like Campbell and Shiller and others, we estimate the dynamic Gordon model using a Vector Auto Regression (VAR). In this VAR model, a vector of state variables, including the cash flow yield of the asset itself, are estimated using the past values of all of the variables in the system. According to the VAR model, the asset's cash flow yield is specified as a linear function of the lagged values of the VAR model's state variables and represents the asset's estimated price formation process.

Having estimated the VAR model, if an asset does not trade during a particular time period, the one-step ahead cash flow yield forecast can then be applied to the asset's observed cash flow to infer the asset's price. In other words, we use the VAR model itself to overcome the lack of trading. Alternatively, if a trade does occur, this market information is taken into account when estimating the VAR model, thereby improving the one-step ahead cash flow yield forecasts of the assets that did not trade. In this way, data on available market transactions are exploited in estimating the prices of non-traded assets. This is analogous to appraisers relying on the market prices of similar properties that recently sold when appraising a commercial property or pricing services using recent transaction prices of fixed income securities with similar characteristics in arriving at a matrix price for a non-traded security.

Our VAR-based methodology has wide applicability in today's financial markets. The methodology is predicated on the period-by-period payment of cash flows by an infrequently traded asset. While some illiquid asset classes, such as OTC stocks and art, do not pay scheduled cash flows, many do. Examples include, commercial real estate, structured credit products and fixed-income securities such as corporate and municipal bonds.

We compare the cash flow yields generated by our methodology to the cash flow yields

implied by pseudo-market prices. In addition, we investigate the extent to which forecasts of cash flow yields based on pseudo-market prices encompass the VAR-based forecasts. That is, we assess if and when the economic factors underlying our VAR model are incorporated in pseudo-market prices. We demonstrate that combining VAR forecasts with pseudo-market prices can improve the predictive content of these proxies, allowing investors to better allocate their investment capital.

Our empirical analysis focuses on two large illiquid asset classes with scheduled cash flows - commercial real estate and commercial mortgage backed securities (CMBS).⁴ As noted by Plazzi, Torous and Valkanov (2010), commercial real estate provides an ideal setting in which to apply the dynamic Gordon model because rents on commercial properties are not discretionary and are paid by tenants as opposed to property managers. Rents are also dependent upon prevailing economic conditions (DiPasquale and Wheaton (1996)). The use of appraisals is widespread in the commercial real estate market. These pseudo-market prices represent an appraiser's opinion of the value of a commercial property. CMBS are a structured financial security collateralized by a pool of commercial mortgages. In a CMBS offering, a series of tranches or certificates are issued forming a waterfall with certificates atop the waterfall having first priority to the promised underlying cash flows while being last to incur losses and are typically AAA rated. While the underlying commercial mortgages may default, CMBS certificates cannot. The CMBS trustee is bound by the deal's Pooling and Servicing Agreement (PSA) to only pass through the mortgage debt service payments to the certificates, thus making values of CMBS certificates particularly sensitive to cash flows received in a particular remittance period. The use of matrix prices is prevalent in valuing CMBS certificates. Unlike appraisals which result from human

⁴NAREIT using data from CoStar estimates the value of U.S. commercial real estate to be \$20.7 trillion as of 2021:Q2. The value of the U.S. CMBS outstanding as of 2021:Q4 stood at \$1.51 trillion according to SIFMA.

decision making, matrix prices are algorithmically determined.

We document that appraisals consistently fail to incorporate the economic factors underlying our VAR model. This conclusion holds at both the individual property as well as portfolio levels. Because we reliably reject the null hypothesis that appraisals encompass VAR-based forecasts, the predictive ability of these pseudo-market prices can be improved by being combined with VAR-based forecasts. We find that appraisals provide particularly poor forecasts of higher valued properties. Value weighted appraisal portfolios in which more weight is given to more valuable properties, for example, when marking to market a portfolio of commercial properties, are particularly noisy and inaccurate.

During the financial crisis, matrix prices of AAA rated CMBS certificates explain almost all of the variation in their corresponding transaction cash flow yields and exhibit almost no price bias. While not as accurate, the explanatory ability of matrix prices of non-AAA rated CMBS certificates during the financial crisis exceeds that of the VAR-based forecasts and easily encompass VAR-based forecasts and underlying economic fundamentals. However, the quality of matrix prices deteriorates dramatically after the financial crisis. Matrix prices of individual non-AAA rated CMBS certificates are now extremely noisy and inaccurate. But VAR-based forecasts show no evidence of such deterioration after the financial crisis. Because matrix prices after the financial crisis are no longer economically disciplined, we can reliably reject the null hypothesis that they encompass VAR-based forecasts. As a result, even the predictive ability of the matrix prices of AAA rated CMBS certificates can be improved during the post-financial crisis period by being combined with VAR-based forecasts.

The plan of this paper is as follows. Based on Campbell and Shiller's dynamic Gordon model, Section 2 puts forward a VAR model for an asset's cash flow yield and its estima-

tion given data on infrequently traded assets with scheduled cash flows. We also discuss methodologies to compare competing cash flow yield forecasts based on our VAR model versus pseudo market prices to assess their predictive adequacy using transaction based cash flow yields of infrequently traded assets. Section 3 discusses our commercial real estate and CMBS data while Section 4 presents our results both at an individual security as well as portfolio level. Section 5 concludes.

2 Methodology

2.1 Dynamic Gordon Model

An asset's cash flow yield measures the cash flow generated by the asset over a period of time relative to its value. According to the dynamic Gordon model, the *log* cash flow yield, denoted by δ , can be expressed as

$$\delta_t = \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}$$

where r denotes the asset's rate of return, Δd denotes the asset's cash flow growth, and ρ is a discounting parameter arising from Campbell and Shiller's log-linearization procedure.

Recognizing that the asset's return, r , can be expressed as the sum of a risk-free rate, i , and a risk-premium, π , the preceding expression becomes:

$$\begin{aligned} \delta_t &= \sum_{j=1}^{\infty} \rho^{j-1} i_{t+j} + \sum_{j=1}^{\infty} \rho^{j-1} \pi_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} \\ &\equiv \mathcal{I}_t + \Pi_t - \mathcal{G}_t \end{aligned}$$

where \mathcal{I}_t represents the long-run risk-free rate, Π_t is the asset's long-run risk premium and

the asset's long-run cash flow growth rate is given by \mathcal{G}_t .

We treat the long-run risk premium as the residual in forecasting an asset's cash flow yield. This assumption allows us to avoid measuring excess returns of assets that trade infrequently and irregularly through time. As a result, we have

$$\begin{aligned}\delta_t &= \hat{\delta}_t + \epsilon_t \\ &= \hat{\mathcal{L}}_t + \hat{\Pi}_t - \hat{\mathcal{G}}_t + \epsilon_t \\ &= \hat{\mathcal{L}}_t - \hat{\mathcal{G}}_t + \mathcal{E}_t.\end{aligned}$$

Therefore, forecasting an asset's cash flow yield requires that we forecast risk-free rates as well as the asset's cash flow growth rate.⁵

2.2 VAR Model

To forecast these state variables, we use a VAR model in which each variable depends of past lags of itself as well as past lags of all the other variables. Let $z_{i,t} = (\delta_{i,t}, r_t, \Delta d_{i,t})'$ be a 3×1 vector of demeaned state variables for the i th cross-sectional asset at time t . For each t in an estimation window, we assume that $z_{i,t}$ is generated by the following VAR(p) specification:

$$z_{i,t} = A_1 z_{i,t-1} + \dots + A_p z_{i,t-p} + e_{i,t} \quad (1)$$

for $i=1, \dots, N$. Here A_j , $j=1, \dots, p$ are 3×3 matrices of slope coefficients and $e_{i,t}$ is a 3×1 vector of disturbances.

⁵The long-run risk premium can be accommodated in this framework by including additional state variables that are hypothesized to be related to the riskiness of the asset's cash flows. In the case of commercial property, Section 4.1, we additionally include a property's NOI volatility, defined as squared NOI growth. In the case of CMBS, Section 4.2, we additionally include returns to the CRSP-Ziman value weighted REIT index, as an observable proxy for a commercial real estate specific risk premium.

Equation (1) imposes the constraint that the time series relation characterizing an asset’s price formation process is the same for each cross-sectional asset within an estimation window. The assumption of homogeneity allows us to pool data in estimation. As has been well documented,⁶ homogeneous panel data estimators perform well out-of-sample when compared to the more parameter consuming heterogeneous estimators.

Letting $z_t = (z'_{1,t}, z'_{2,t}, \dots, z'_{N,t})'$ denote the stacked $3 \cdot N \times 1$ vector of demeaned state variables for the N cross-sectional assets, the pooled VAR(p) model can be written as:

$$z_t = \mathcal{A}_1 z_{t-1} + \dots + \mathcal{A}_p z_{t-p} + e_t \quad (2)$$

where \mathcal{A}_i is a $3 \cdot N \times 3 \cdot N$ block-diagonal matrix with A_i on its diagonal and e_t is a $3 \cdot N \times 1$ vector of disturbances. Since this VAR(p) model is in the form of a seemingly unrelated regression (SUR) where each variable is explained by the same regressors, each equation can be estimated separately by ordinary least squares (OLS).

2.3 Estimation

We use a rolling forecast scheme and iteratively estimate equation (2). The asset’s estimated price formation process, therefore, varies over time. A time-varying price formation process is economically reasonable as investors make different use of prevailing information as economic conditions change.

In the initial estimation window of length ω periods, from $t=1$ to $t=\omega$, if any cross-sectional asset did not trade during a particular period, we have no choice but to train the VAR(p) model using its cash flow yield based on a pseudo-market price such as, for example, an appraisal or matrix price. If a cross-sectional asset or “similar” asset did trade,

⁶See Baltagi (2008).

the transaction-based cash flow yield is used. By similar asset we mean, for example, in the case of commercial real estate, a building of the same property type sold immediately nearby or, in the case of a CMBS certificate or other fixed income security, a security of the same type issued in the same year with the same original credit rating. Observed interest rates and cash flow growth of all of the cross-sectional assets are relied upon throughout. Having estimated the VAR(p) model over the initial estimation window, one-step ahead forecasts of the cash flow yields of all of the cross-sectional assets are generated for period $t=\omega+1$.

Moving forward one period, we once again estimate the VAR(p) model but now over the subsequent estimation window of length ω periods, from $t=2$ to $t=\omega+1$. However, in training the VAR over this as well as all subsequent estimation windows, we distinguish between whether or not a cross-sectional asset traded in the incremental period $t=\omega+1$. If a cross-sectional asset or similar asset did trade, we use this transaction-based cash flow yield. But if a cross-sectional asset did not trade, we use the one-step ahead forecast of its cash flow yield previously generated for period $t=\omega+1$. Once again, the interest rate and cash flow growth of all of the cross-sectional assets observed in period $t=\omega+1$ are used. One-step ahead cash flow yield forecasts of all of the cross-sectional assets are now generated for period $t=\omega+2$.

Proceeding iteratively in this fashion, we estimate the VAR(p) model over successive overlapping estimation windows of length ω periods through the end of period T . Each successive estimation is augmented by the cross-sectional assets' state variables, including their cash flow yields, measured over the incremental period added. Our methodology is distinguished by the fact that if a cross-sectional asset did not trade in this incremental period, we use the asset's one-step ahead cash flow yield just forecasted. For example,

without loss of generality, consider the estimated cash flow yield from a VAR(1) specification of the i th cross-sectional asset which did not trade during period $t+1$. If the asset also did not trade in period t , we have:

$$\hat{\delta}_{i,t+1} = a_{\delta\delta}^t \hat{\delta}_{i,t} + a_{\delta r}^t r_t + a_{\delta\Delta d}^t \Delta d_{i,t} \quad (3)$$

where the $\{a_{i,j}^t\}$ are the estimated slope coefficients of the VAR(1) model based on data from the estimation window through period t , r_t and $\Delta d_{i,t}$ are, respectively, the interest rate and cash flow growth of the i th asset observed in period t , while $\hat{\delta}_{i,t}$ is the i th asset's previously forecasted cash flow yield for period t generated using data from the estimation window through period $t-1$. Equation (3) can immediately be seen as an iterated forecast based on the assumed VAR model. However, unlike extant VAR-based iterative forecasting, the estimated VAR model is now updated to reflect the arrival of new information during the incremental t th period, including any market transactions in the cross-section of assets, and so can be expected to improve out-of-sample forecasting.

2.4 Assessing the Adequacy of Pseudo-Market Prices

2.4.1 Individual Assets

We compare the out-of-sample (*OOS*) accuracy of VAR-based forecasts, f_{var} , in predicting an asset's transaction-based cash flow yield to forecasts based on pseudo-market prices, f_{pseudo} . We also assess whether VAR-based forecasts embody useful information absent in pseudo-market forecasts. Evidence that forecasts based on pseudo-market prices fail to encompass VAR-based forecasts identify when underlying economic fundamentals are not fully reflected in pseudo-market prices.

To do so, note that competing *OOS* cash flow yield forecasts under our rolling forecast scheme are available starting at period $t=\omega+1$ and culminating at period $t=T$. In our applications, time periods are measured in months, quarters, or half-years, corresponding to the frequency with which asset cash flows are typically reported. So even though an asset trades infrequently, it is to be expected that a number of cross-sectional assets transact within these reporting periods. This then allows us to construct transaction-based cash flow yields that competing forecasts can be compared to. We denote the resultant *OOS* forecast error for cross-sectional asset i at period t using pseudo-market prices by $\xi_{pseudo\ i,t}$ and using our VAR model by $\xi_{var\ i,t}$.

For a particular asset type and given sample period, we can now calculate how much of the corresponding variation in individual assets' transaction-based cash flow yields is explained by the variation in competing *OOS* forecasts:

$$OOS\ R_{pseudo}^2 = 1 - \frac{MSE_{pseudo}}{MSE_{historical}} \quad \text{and} \quad OOS\ R_{var}^2 = 1 - \frac{MSE_{var}}{MSE_{historical}}$$

where $MSE_{historical}$ represents the mean-squared error in explaining transaction-based cash flow yields using the overall average transaction-based cash flow yield while MSE_{pseudo} and MSE_{var} are corresponding mean-squared errors based on pseudo-market prices and the VAR model, respectively. The more of an improvement in predicting transaction-based cash flow yields over the historical mean, the closer to one the *OOS* R^2 statistic becomes. If the prediction does no better than the historical mean, the statistic is zero and is negative if the prediction is worse.

Given the competing forecasts, encompassing tests can be used to assess the informativeness of forecasts of individual assets' transaction-based cash flow yields based on pseudo-market prices. In particular, forecasts based on pseudo-market prices encompass

VAR-based forecasts if for a convex combination of these forecasts

$$f_c = (1-\lambda)f_{pseudo} + \lambda f_{var} \quad 0 \leq \lambda \leq 1$$

we cannot statistically reject

$$H_0: \lambda=0.$$

Alternatively, the VAR model embodies useful information absent in pseudo-market prices under the alternative that $\lambda > 0$.

We estimate λ for a particular asset type and given sample period by numerically determining its value which minimizes the corresponding sum of squared combined forecast errors:

$$\sum_{t=R}^S \sum_{i=1}^{I_t} ((1-\lambda)\xi_{pseudo\ i,t} + \lambda\xi_{var\ i,t})^2$$

given I_t transactions during time period t and sample period $\{t|R \leq t \leq S\}$. The larger this value of λ , the greater the weight of the VAR model and the lesser the weight of pseudo-market prices in optimally forecasting transaction-based cash flow yields in this sample. The uncertainty surrounding the λ estimate can be assessed by block bootstrapping across all transactions during the sample period.

2.4.2 Portfolios

Institutional investors typically hold portfolios of assets. These portfolios are often marked to market on a periodic basis to update their values in the face of changing market conditions. If a portfolio contains infrequently traded assets then pseudo-market prices of these assets, for example, appraisals or matrix prices, will often be relied upon when marking

to market the portfolio's value. We now turn attention to investigating the adequacy of pseudo-market prices in the context of portfolios of infrequently traded assets. To the extent that pseudo-market prices of individual assets idiosyncratically deviate from corresponding transaction prices, diversification may improve their accuracy and ability to encompass underlying economic fundamentals.

Given forecast errors of the cash flow yields of the individual assets in a portfolio, the portfolio-weighted average of these forecast errors can be formed on a period-by-period basis. By diversifying across cross-sectional forecast errors, we now have a time series of portfolio *OOS* forecast errors using pseudo-market prices, $\{\xi_{pseudo t}\}$, and using our VAR model, $\{\xi_{var t}\}$.

Given the time series of competing forecast errors, we evaluate forecast accuracy using the model-free approach of Diebold and Mariano (1995) (DM). The DM test is robust to potentially contemporaneously correlated, serially correlated and non-normal forecast errors. Define the loss differential d_t by

$$d_t = \xi_{pseudo t}^2 - \xi_{var t}^2$$

and we test the equality of the expected squared errors of the competing forecasts

$$H_0: E(d_t)=0$$

using DM's test statistic, given by the ratio of the sample mean of the difference in squared forecast errors \bar{d} to its estimated standard error, modified by the Harvey, Leybourne and Newbold (1997) finite sample correction.

We can also robustly test whether forecasts of a portfolio of cash flow yields implied by

pseudo-market prices encompass portfolio forecasts based on our VAR methodology.

To estimate λ and derive the appropriate test statistic, note that the value of λ which minimizes the combined portfolio forecast's mean-squared error satisfies

$$\lambda^* = \frac{E(\xi_{pseudo t}(\xi_{pseudo t} - \xi_{var t}))}{E(\xi_{pseudo t} - \xi_{var t})^2}.$$

This implies that we can estimate λ by running the following time-series regression

$$\xi_{pseudo t} = \lambda(\xi_{pseudo t} - \xi_{var t}) + \xi_{combined t}$$

where $\xi_{combined t}$ denotes the error of the combined portfolio forecast. The combined portfolio forecast will have a smaller mean-squared error than the portfolio forecast based on pseudo-market prices unless the covariance of $\xi_{pseudo t}$ and $\xi_{pseudo t} - \xi_{var t}$ is zero. It follows that the null hypothesis of forecast encompassing can be expressed as

$$H_0: E(d_t) = 0$$

where the differential d_t is given by

$$d_t = \xi_{pseudo t}(\xi_{pseudo t} - \xi_{var t})$$

and can be robustly tested using the approach of Diebold and Mariano with Harvey, Leybourne, and Newbold's finite sample correction.

3 Data

3.1 Commercial Real Estate

The commercial real estate data comes from the National Council of Real Estate Investment Fiduciaries (NCREIF) which maintains an extensive database of commercial property holdings that covers the majority of privately held institutional grade commercial real estate in the U.S. The NCREIF database provides, among other items, the following transaction-level data: listings of properties held in each fund manager’s portfolio, transaction prices and sale dates for properties, quarterly income (NOI) as well as quarterly property appraisals.

Appraisals in commercial real estate are conducted by human appraisers according to accepted practices and standards. Appraisals are not forecasts *per se* but rather reflect the appraiser’s current assessment of a property’s value. Nonetheless, much previous research has investigated the accuracy of appraisals in predicting subsequent sale prices.⁷

Because NCREIF data is reported quarterly, we compare a cash flow yield based upon the agreed upon sales price to the cash flow yield implied by the appraisal lagged one-quarter prior to the transaction. Similarly, we rely on one-quarter ahead VAR forecasts of cash flow yields made one-quarter prior to any property sale.

The details of the commercial property sales process, however, should be recognized when evaluating the forecasting ability of appraisals. In particular, a property sale is typically accompanied by a lengthy due diligence period that can span a number of quarters. So when a sale is pending and contract terms become known, appraisers will often rely on the sales price in the quarter preceding the transaction in lieu of conducting an actual

⁷For example, Webb (1994) and Fisher, Miles and Webb (1999), in addition to Cannon and Cole (2011).

appraisal.⁸ This will have the effect of increasing the accuracy of appraisals and improve their forecasting performance relative to the VAR forecasts. But even with the possibility that appraisers rely on actual sales prices, our subsequent analysis shows that VAR forecasts still provide valuable information that improve the forecast accuracy of appraisals.⁹

3.2 Commercial mortgage backed securities

Commercial mortgage backed securities (CMBS) are fixed income securities collateralized by a pool of commercial mortgages. Cash flows play an important role in the pricing of CMBS because of the senior/subordinate waterfall typically relied upon in CMBS deals. In particular, debt service from the underlying pool of mortgages is passed through to the tranches or certificates of a CMBS in sequential order. Principal and interest payments are paid first to the most senior certificate, typically AAA rated, and then subsequently to the more subordinate non-AAA rated certificates. On the other hand, losses on the underlying pool of commercial mortgage loans, arising from delinquencies and defaults, are assigned in reverse order with the most junior certificate receiving losses first.

Pricing information on CMBS trades is not disseminated through the TRACE system.¹⁰ Fortunately, insurance companies are required to report the transaction prices of each security they trade (Merrill, Nadauld, Stulz and Sherlund (2021)) including CMBS. As a result, we obtain transaction prices of CMBS certificates from insurance companies

⁸For example, Cannon and Cole (2011) report that in their NCREIF sample approximately one-half of appraised values in the quarter prior to a sale are exactly equal to the sales price. In our NCREIF sample, 36.7% of appraised values in the quarter prior to a sale are within \$10,000 of the sales price.

⁹Our analysis does not discard any one-quarter lagged appraisals as we cannot confirm whether the appraiser actually relied on a known sales price. Also, we do not follow Cannon and Cole and rely on appraisals lagged two quarters prior to any property sale to ensure they do not contain information about upcoming transactions. Comparing two-quarter lagged appraisals to VAR forecasts made only one quarter prior to a transaction would disadvantage appraisals as appraisers would not have access to as up-to-date information as the VAR model.

¹⁰See FINRA Rule 6750.

through regulatory filings made available by the National Association of Insurance Commissioners (NAIC). The Thomson Reuters Eikon database provides matrix prices of CMBS certificates.

Cash flow information on CMBS is updated monthly via remittance reports. We obtain the information in these remittance reports from Trepp. Matrix prices, on the other hand, change at a higher frequency. In fact, it is not uncommon for matrix prices to be updated at a daily frequency. Therefore, we compare the cash flow yield based on an observed CMBS transaction price to the corresponding cash flow yield implied by the latest matrix price available prior to the transaction. In a vast majority of cases, for both AAA rated as well as non-AAA rated certificates, the latest matrix price is observed one day prior to a transaction.¹¹ By contrast, since our CMBS VAR model is implemented on a monthly basis, we rely on one-month ahead VAR cash flow yield forecasts made at the end of the month prior to any CMBS certificate sale.

4 Results

4.1 NCREIF Properties

4.1.1 Individual Properties

In Table 1, we see that appraisals capture $OOS R^2_{appraisal}=43.47\%$ of the variation in transaction cash flow yields across all individual NCREIF properties while VAR-based forecasts capture only $OOS R^2_{var}=29.71\%$. This result, however, may reflect the possibility that some appraisers relied on actual sales prices. We show in the Internet Appendix

¹¹In our overall sample, the delay between the date when a matrix price was last updated and the trade date for AAA rated certificates is more than 1 day for 18.3% of trades and is more than 3 days for 1.0% of trades. For non-AAA rated certificates, the delay is more than 1 day for 16.8% of trades and is more than 3 days for 1.0% of trades.

that the predictive ability of appraisals across all individual properties decreases to only $OOS R^2_{appraisal}=19.83\%$ when we rely on appraisals lagged two-quarters prior to transactions to ensure that appraisals do not contain information about upcoming transactions.

Nevertheless, the fitted λ value of 0.3913 is statistically different from zero at the 1% significance level, indicating that appraisals lagged one-quarter prior to transactions do not encompass our VAR-based forecasts and that the economic fundamentals underlying the VAR-based forecasts add useful information to predictions based solely on these appraisals. The extent to which the optimal combined forecast improves is demonstrated by the $OOS R^2_{combined}$ of 58.72%. This is a substantial improvement, more than fifteen percentage points, over the predictive power of either individual forecast

Across all individual NCREIF properties, appraisals overvalue¹² properties by an average of 1.57 percentage points. By contrast, the VAR-based forecasts overvalue by only 0.97 percentage points.

Table 1 also compares competing cash flow yield forecasts for different property sectors. While the $OOS R^2$ of the appraisals exceed that of the VAR-based predictions throughout¹³, the fitted λ values are statistically significantly different from zero across all property types, ranging from 0.2927 for apartments to 0.4352 for industrial, consistent with appraisals not encompassing VAR-based forecasts. As expected, for all property sectors, the resultant $OOS R^2_{combined}$ substantially exceed the $OOS R^2$ of the individual forecasts. Appraisals tend to undervalue apartments and retail, while VAR-based forecasts overvalue all property types except apartments.

¹²We calculate the percentage bias in a property's forecasted price using the property's forecasted cash flow yield and transaction cash flow yield given the property's cash flow in the quarter of the transaction.

¹³The predictive ability of appraisals decreases and is less than that of the VAR forecasts for each property type when we rely on two-quarter lagged appraisals to ensure that appraisals do not contain information about upcoming transactions. See the Internet Appendix.

4.1.2 Portfolios of Properties

We next turn our attention to comparing VAR-based forecasts to appraisals for portfolios of properties. We first form equal weighted portfolios in competing forecasts of cash flow yields each quarter for properties which transacted in that quarter. Doing so means that the forecast error of a property with a small appraised value is treated no differently than the forecast error of a property with a large appraised value.

In Panel A of Table 2, we see that diversification improves the accuracy and explanatory power of appraisals. While the explanatory power of the VAR-based portfolio forecasts also improves, appraisals capture more of the variation in the equal weighted portfolio of transaction cash flow yields than do our VAR-based forecasts for all properties as well as each individual property type. Relying on the $DM_{accuracy}$ test, we see no evidence that our VAR-based portfolio forecasts are more accurate than appraisal portfolio forecasts. However, the $DM_{encompassing}$ test shows that VAR-based portfolio forecasts are not encompassed for all property types and each individual property type except industrial. The resultant time series estimates of λ range from 0.2657 for retail to 0.3966 for apartments. Using these single time series λ estimates to combine corresponding quarterly appraisal portfolio forecasts with quarterly VAR-based portfolio forecasts, denoted as the $combined_{\#1}$ forecast, captures substantially more of the variation in the equal weighted portfolio of transaction cash flow yields. For example, $OOS R^2_{combined_{\#1}}$ is 59.02% for offices as compared to $OOS R^2_{appraisal}$ of only 45.65%. Alternatively, the $combined_{\#2}$ forecast relies on varying λ values that minimize each quarter's sum of squared combined forecast errors. Taking into account this cross-sectional variation improves forecasts with $OOS R^2_{combined_{\#2}}$ exceeding $OOS R^2_{combined_{\#1}}$ for equal weighted portfolios of all properties and each individual property type.

The message, however, is decidedly different in Panel B of Table 2 where VAR-based forecasts are compared to appraisals for value weighted portfolios. Now more weight is given to forecasting transaction cash flow yields of properties with larger appraised values. These results, as opposed to the results for equal weighted portfolios, are more relevant to institutional investors who, for example, mark to market portfolios of infrequently traded commercial properties.

Our evidence here is consistent with appraisals providing poorer forecasts of transaction cash flow yields for higher valued properties. Appraisals of these properties rely less on economic fundamentals, diminishing their accuracy and explanatory power. For example, VAR-based forecasts now capture more of the variation in the value weighted portfolio of transaction cash flow yields than do appraisals for all properties as well as each individual property sector. The noisy and inaccurate nature of appraisal portfolio forecasts is evidenced by the negative values of $OOS R^2_{appraisal}$ for value weighted portfolios of all properties as well as each property type except apartments. VAR-based forecasts, by contrast, capture comparable variation in value weighted portfolios of transaction cash flow yields as in the equal weighted portfolios. The $DM_{accuracy}$ test provides evidence that the VAR model more accurately forecasts value weighted portfolios of transaction cash flow yields for all properties as well as industrial and office properties. Once again, the $DM_{encompassing}$ test rejects throughout the null hypothesis that appraisals encompass VAR-based forecasts and their reliance on underlying economic fundamentals. The resultant time series estimates of λ are now closer to one indicating that the optimal combined forecasts rely more on the VAR-based forecasts. Finally, $OOS R^2_{combined\#1}$ exceeds $OOS R^2_{combined\#2}$ for value weighted portfolios of transaction cash flow yields for all properties and apartments and industrials. Intuitively, to the extent appraisals of higher valued properties that transact in a given quarter are associated with very large forecast errors, the resultant extreme

cross-sectional variation in forecast errors deteriorates the forecasting properties of the *combined*_{#2} forecasts.

4.2 NAIC CMBS certificates

4.2.1 Individual CMBS certificates

Panel A of Table 3 shows that for the entire sample period, matrix prices explain much more of the variation in transaction cash flow yields of individual AAA rated CMBS certificates, $OOS R^2_{matrix}=78.91\%$, than VAR-based forecasts, $OOS R^2_{var}=39.24\%$. The fitted value of $\lambda=0.2662$, while statistically different from zero at the 1% significance level, implies that the resultant combined forecast, $OOS R^2_{combined}=90.96\%$, places most of its weight on matrix prices.

By contrast, matrix prices do a particularly poor job in explaining the transaction cash flow yields of individual non-AAA rated CMBS certificates over the entire sample period. Matrix prices of these certificates are noisy and inaccurate, as evidenced by a negative $OOS R^2_{matrix} = -37.38\%$. The corresponding fitted value of $\lambda=0.6904$ implies that the resultant combined forecast, $OOS R^2_{combined}=48.55\%$, now places most of its weight on the VAR-based forecasts.

In light of these forecasting difficulties, it is not surprising that VAR-based forecasts capture more of the variation in the transaction cash flow yields of the overall sample, both AAA rated as well as non-AAA rated CMBS certificates, $OOS R^2_{var}=33.91\%$ versus $OOS R^2_{matrix}=13.68\%$. The $\lambda=0.5897$ value fitted for the overall sample across the entire sample period is consistent with matrix prices not encompassing VAR-based forecasts and underlying economic fundamentals.

Concentrating on the entire sample period, however, masks important differences in

the properties of matrix prices between the financial crisis, 2007:01 to 2008:12, and the post financial crisis period, 2009:01 to 2013:12. As is evident from Panels B and C of Table 3, matrix prices do a much better job during the financial crisis, for both AAA rated as well as non-AAA rated CMBS certificates. The fitted λ values here are, for the most part, indistinguishable from zero indicating that matrix prices encompass the VAR-based forecasts and underlying economic fundamentals. Matrix prices explain more than twice as much of the variation in the transaction cash flow yields of individual AAA rated certificates during the financial crisis: $OOS R_{matrix}^2=89.67\%$ versus $OOS R_{var}^2=42.53\%$. Both matrix prices and the VAR model are less accurate in forecasting transaction cash flow yields of non-AAA rated certificates during the financial crisis but matrix prices have more forecasting ability here as well: $OOS R_{matrix}^2=44.68\%$ versus $OOS R_{var}^2=30.02\%$. The bias of matrix prices during the financial crisis is practically non-existent for both AAA rated and non-AAA rated CMBS certificates. This contrasts with VAR-based forecasts which exhibit larger biases, especially in the case of non-AAA rated certificates which the VAR model overprices by almost 3% during the financial crisis.

However, the properties of matrix prices deteriorate dramatically after the financial crisis, especially for individual non-AAA rated CMBS certificates. The fitted λ values can be seen to be statistically different from zero at the 1% significance level throughout, indicating that matrix prices now do not encompass VAR-based forecasts and underlying economic fundamentals. While matrix prices still explain more of the variation in the transaction cash flow yields of AAA rated certificates, they are noisy and inaccurate in the case of non-AAA rated certificates as evidenced by a negative $OOS R_{matrix}^2 = -42.95\%$.¹⁴

¹⁴These results are not due to matrix prices of non-AAA rated certificates being stale when compared to those of AAA rated certificates. In our post financial crisis sample, the delay between the date when a matrix price was last updated and the trade date is more than 1 day for 17.6% of AAA trades as compared to 16.3% of non-AAA trades. The delay is more than 3 days for 0.6% of both AAA and non-AAA trades in the post financial crisis period.

The pricing bias of the VAR model for all certificates is smaller than the pricing bias of matrix prices during the post financial crisis period. This reflects the fact that matrix prices undervalue individual non-AAA certificates during the post financial crisis period by more than 5% while still valuing individual AAA rated certificates almost as well as during the financial crisis.

The fact that matrix prices, especially of non-AAA rated certificates, no longer reflect underlying economic fundamentals in the post financial crisis period suggests that the financial crisis prompted a change in the matrix pricing algorithm. Despite the poor performance of the matrix prices of non-AAA rated certificates after the financial crisis, adding the predictive content of the VAR forecasts significantly improves predictive ability. The combined model now explains almost half of the variation in the transaction cash flow yields of individual non-AAA rated CMBS certificates after the financial crisis, $OOS R^2_{combined}=47.00\%$.

4.2.2 Portfolios of CMBS certificates

In Panel A of Table 4, VAR-based forecasts are compared to matrix prices for an equal weighted portfolio of CMBS transaction cash flow yields. Matrix prices are remarkably accurate in forecasting transaction cash flow yields of an equal weighted portfolio of AAA rated CMBS certificates, $OOS R^2_{matrix}=95.04\%$, as compared to the VAR model's $OOS R^2_{var}=66.91\%$. The null hypothesis that matrix prices encompass VAR forecasts cannot be rejected here and combining both does little to improve forecasting ability.

However, matrix prices do a particularly poor job in forecasting transaction cash flow yields of an equal weighted portfolio of non-AAA rated CMBS certificates, $OOS R^2_{matrix}=-49.65\%$. In comparison, the VAR model captures almost half of the observed variation,

$OOS R_{var}^2=49.93\%$. The $DM_{accuracy}$ test provides reliable evidence that the VAR model more accurately forecasts the equal weighted portfolio of non-AAA rated transaction cash flow yields. The null hypothesis that matrix prices encompass VAR-based forecasts and their reliance on underlying economic fundamentals is rejected by the $DM_{encompassing}$ test. The resultant time series estimate of $\lambda = 0.8106$ can be used to combine the complimentary information in matrix prices and VAR-based forecasts to improve the forecasting of non-AAA rated transaction cash flow yields, $OOS R_{combined\#1}^2=57.35\%$. However, we can do even better by combining these forecasts by relying on λ values that minimize each month's sum of squared combined forecast errors. Taking into account cross-sectional variation improves forecasts of the equal weighted portfolio of transaction cash flow yields of non-AAA rated CMBS certificates to $OOS R_{combined\#2}^2=78.45\%$.

Panel B of Table 4 considers value weighted portfolios of CMBS transaction cash flow yields. Value weights here are based on a CMBS certificate's principal balance outstanding as of the distribution date immediately preceding the transaction date.

Since the principal balance of CMBS offerings is primarily in AAA rated certificates, value weighted portfolios of all CMBS certificates will be dominated by the included AAA rated certificates. This can be seen in Panel B of 4 where the results for the value weighted portfolio of all CMBS certificates are almost indistinguishable from the results for the value weighted portfolio of AAA rated CMBS certificates. In both cases, matrix prices explain almost all of the variation in the value weighted portfolio of transaction cash flow yields and encompass the corresponding VAR-based forecasts. Combining VAR-based forecasts with matrix prices offers negligible improvement in forecasting ability.

But, as expected, matrix prices provide inaccurate and noisy forecasts of a value weighted portfolio of non-AAA rated CMBS transaction cash flow yields. VAR-based fore-

casts capture much more of the variation in this value weighted portfolio, $OOS R_{var}^2=53.10\%$ versus $OOS R_{matrix}^2= - 20.48\%$. The VAR model more accurately forecasts the value weighted portfolio of non-AAA rated transaction cash flow yields according to the $DM_{accuracy}$ test and the $DM_{encompassing}$ test rejects the null hypothesis that matrix prices encompass VAR-based forecasts and their reliance on underlying economic fundamentals. Combining VAR-based forecasts with matrix prices by relying on λ values that minimize each month's sum of squared combined forecast errors improves forecasts of the value weighted portfolio of transaction cash flow yields of non-AAA rated CMBS certificates to $OOS R_{combined\#2}^2=73.05\%$.

5 Conclusions

Investors in thinly traded markets often rely on pseudo-market prices, such as appraisals and matrix prices, to approximate the market values of their portfolios. In this paper, we compare corresponding cash flow yields based on pseudo-market prices to the results of our VAR-based methodology to gauge the extent to which pseudo-market prices are grounded in economic fundamentals.

Appraisals of institutional-grade commercial properties consistently fail to fully incorporate the fundamental economic factors underlying our simple VAR model. Forecast accuracy can be significantly improved by combining appraisals with VAR-based forecasts. Appraisers being human make mistakes and we find that diversification, in general, improves the predictive performance of appraisals at the portfolio level. However, the predictive performance of appraisals for value weighted portfolios, relevant to investors marking to market a portfolio of commercial properties, are inaccurate and noisy reflecting the fact that appraisal accuracy deteriorates for higher valued property transactions.

Algorithmically determined matrix prices more accurately predict transaction prices of AAA rated CMBS certificates as opposed to certificates rated non-AAA at origination. Rather than deteriorating during the financial crisis, matrix prices of both AAA and non-AAA rated CMBS certificates performed remarkably well and fully encompassed the information contained in our VAR model. However, the predictive content of these matrix prices, especially matrix prices of non-AAA rated certificates, worsened and no longer reflected economic fundamentals after the financial crisis suggesting a change in the matrix pricing algorithm prompted by the financial crisis. The predictive ability of matrix prices after the financial crisis is significantly improved when combined with VAR-based forecasts which capture the effects of economic fundamentals devoid in prevailing matrix prices. As a result, investors can more accurately mark to market portfolios of these thinly traded assets and better allocate investment capital.

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Table 1: Individual NCREIF Properties

We tabulate R^2 statistics of out-of-sample cash flow yield forecasts using one-quarter lagged appraisals (OOS $R^2_{appraisal}$) and the VAR model (OOS R^2_{VAR}) for all individual NCREIF properties, only NCREIF apartments, only NCREIF industrial properties, only NCREIF office properties, and only NCREIF retail properties over the sample period 1978:IV to 2013:II. The weight λ placed on the VAR forecast that minimizes the sum of squared combined out-of-sample cash flow yield forecasts is also tabulated along with the resultant combined forecast's R^2 statistics (OOS $R^2_{combined}$). We assess the uncertainty surrounding the λ estimates by block bootstrapping across all transactions during the sample period. We also provide the median percentage bias in an individual property's forecasted price using appraisals and the VAR model.

	all	apartments	industrial	office	retail
OOS $R^2_{appraisal}$	43.47%	51.02%	27.97%	45.92%	54.08%
OOS R^2_{VAR}	29.71%	29.40%	22.99%	19.22%	32.47%
λ	0.3913**	0.2927**	0.4352**	0.3609**	0.3628**
OOS $R^2_{combined}$	58.72%	63.88%	52.27%	69.07%	73.29%
Appraisal price bias	1.56%	-3.24%	0.41%	0.79%	-3.49%
VAR price bias	0.97%	-8.49%	5.59%	2.50%	3.71%

** denotes statistical significance at the 1% level.

Table 2: Portfolios of NCREIF Properties

We form equal weighted portfolios (Panel A) and value weighted portfolios using appraised property values (Panel B) of out of sample cash flow yield forecasts. We tabulate resultant R^2 statistics using one-quarter lagged appraisals (OOS $R_{appraisal}^2$) and the VAR model (OOS R_{VAR}^2) for portfolios of all individual NREIF properties, only NCREIF apartments, only NCREIF industrial properties, only NCREIF office properties, and only NCREIF retail properties over the sample period 1978:IV to 2013:II. We also provide Diebold-Mariano robust test statistics of forecast encompassing ($DM_{encompassing}$) and equality of forecast accuracy ($DM_{accuracy}$) along with the time series estimates of the weight λ of the VAR forecast in the optimal combined forecast. We combine appraisals and VAR forecasts by using the single time series estimate of λ , this combined forecast's goodness of fit measured by OOS $R_{combined-1}^2$, and by using varying estimates of λ that minimize each month's sum of squared combined forecast errors, this combined forecast's goodness of fit measured by OOS $R_{combined-2}^2$.

Panel A: Equal Weighted					
	all	apartments	industrial	office	retail
OOS $R_{appraisal}^2$	74.74%	58.37%	54.57%	45.65%	66.00%
OOS R_{var}^2	50.42%	50.48%	30.00%	13.85%	39.25%
λ	0.2878	0.3966	0.3482	0.3684	0.2657
$DM_{encompassing}$	0.5004	1.9625 ^o	0.9848	1.4563 ^o	1.6885*
$DM_{accuracy}$	-2.2194	-0.7637	-1.4543	-0.9832	-1.3656
OOS $R_{combined-1}^2$	79.46%	64.37%	64.19%	59.02%	70.03%
OOS $R_{combined-2}^2$	83.58%	79.54%	76.12%	80.01%	77.78%

Panel B: Value Weighted					
	all	apartments	industrial	office	retail
OOS $R_{appraisal}^2$	-16.85%	22.18%	-40.77%	-48.90%	-6.58%
OOS R_{var}^2	51.99%	46.63%	37.00%	26.31%	35.18%
λ	0.7648	0.6618	0.7524	0.7155	0.6965
$DM_{encompassing}$	2.2681*	1.8861*	2.5913**	2.5823**	2.6696**
$DM_{accuracy}$	1.7762*	0.1635	1.7177*	1.5632 ^o	1.1854
OOS $R_{combined-1}^2$	59.18%	55.27%	46.45%	40.43%	44.96%
OOS $R_{combined-2}^2$	44.82%	47.79%	39.50%	62.01%	29.60%

** denotes statistical significance at the 1% level, * at the 5% level, and ^o at the 10% level.

Table 3: Individual CMBS Certificates

We tabulate R^2 statistics of out-of-sample cash flow yield forecasts using matrix prices (OOS R^2_{matrix}) and the VAR model (OOS R^2_{VAR}) for all individual CMBS certificates, only CMBS certificates rated AAA at their origination, and only CMBS certificates not rated AAA at their origination over the sample period 2007:1 to 2013:12, the financial crisis period 2007:1 to 2008:12, and the post-financial crisis period 2009:1 to 2013:12. The weight λ placed on the VAR forecast that minimizes the sum of squared combined out-of-sample cash flow yield forecasts is also tabulated along with the resultant combined forecast's R^2 statistics (OOS $R^2_{combined}$). We assess the uncertainty surrounding the λ estimates by block bootstrapping across all transactions during the sample period. We also provide the median percentage bias in an individual CMBS certificate's forecasted price using matrix prices and the VAR model for all individual CMBS certificates, only AAA rated CMBS certificates, and only non-AAA rated CMBS certificates over the full sample period, the financial crisis period, and the post-financial crisis period.

Panel A: Full Sample Period			
	all certificates	AAA rated	non-AAA rated
OOS R^2_{matrix}	26.22%	78.91%	-37.38%
OOS R^2_{var}	32.99%	39.24%	15.18%
λ	0.5330**	0.2662**	0.6904**
OOS $R^2_{combined}$	67.48%	90.96%	48.55%

Panel B: Sample Subperiods			
	all certificates	AAA rated	non-AAA rated
Financial Crisis 2007:01-2008:12			
OOS R^2_{matrix}	79.06%	89.67%	44.68%
OOS R^2_{var}	43.65%	42.53%	30.02%
λ	0.2004	0.1593 ^o	0.3533
OOS $R^2_{combined}$	87.21%	95.25%	68.79%
Post-Financial Crisis 2009:01-2013:12			
OOS R^2_{matrix}	19.34%	76.07%	-42.95%
OOS R^2_{var}	30.81%	36.49%	13.59%
λ	0.5527**	0.2821**	0.6966**
OOS $R^2_{combined}$	64.81%	89.81%	47.00%

Panel C: Pricing Bias			
	full sample	financial crisis 2007:01-2008:12	post-financial crisis 2009:01-2013:12
matrix price bias			
all certificates	-0.49%	-0.26%	-0.76%
AAA rated	-0.07%	0.03%	-0.07%
non-AAA rated	-1.93%	-0.50%	-5.50%
VAR price bias			
all tranches	0.26%	0.56%	-0.18%
AAA rated	-0.39%	-0.45%	-0.68%
non-AAA rated	1.99%	2.97%	0.12%

** denotes statistical significance at the 1% level and ^o at the 10% level.

Table 4: Portfolios of CMBS Certificates

We form equal weighted portfolios (Panel A) and value weighted portfolios using outstanding CMBS certificate principal balance (Panel B) of out of sample cash flow yield forecasts. We tabulate resultant R^2 statistics using matrix prices (OOS R^2_{matrix}) and the VAR model (OOS R^2_{VAR}) for portfolios of all individual CMBS certificates, only CMBS certificates rated AAA at their origination, and only CMBS certificates not rated AAA at their origination over the sample period 2007:1 to 2013:12. We also provide Diebold-Mariano robust test statistics of forecast encompassing ($DM_{encompassing}$) and equality of forecast accuracy ($DM_{accuracy}$) along with the time series estimates of the weight λ of the VAR forecast in the optimal combined forecast. We combine matrix prices and VAR forecasts by using the single time series estimate of λ , this combined forecast's goodness of fit measured by OOS $R^2_{combined-1}$, and by using varying estimates of λ that minimize each quarter's sum of squared combined forecast errors, this combined forecast's goodness of fit measured by OOS $R^2_{combined-2}$.

Panel A: Equal Weighted			
	all tranches	AAA rated	non-AAA rated
OOS R^2_{matrix}	65.51%	95.04%	-49.68%
OOS R^2_{var}	64.31%	66.91%	49.93%
λ	0.4924	0.1927	0.8310
$DM_{encompassing}$	2.1863*	-0.7671	3.5207**
$DM_{accuracy}$	-0.0936	-3.9686	2.5591**
OOS $R^2_{combined-1}$	84.66%	96.74%	54.23%
OOS $R^2_{combined-2}$	92.46%	98.20%	78.45%

Panel B: Value Weighted			
	all tranches	AAA rated	non-AAA rated
OOS R^2_{matrix}	97.67%	97.91%	-20.48%
OOS R^2_{var}	76.21%	76.23%	53.10%
λ	0.1595	0.1496	0.8106
$DM_{encompassing}$	-1.9829	-2.1665	3.0604**
$DM_{accuracy}$	-4.5633	-4.6666	2.3939**
OOS $R^2_{combined-1}$	98.47%	98.60%	57.35%
OOS $R^2_{combined-2}$	98.29%	98.69%	73.05%

** denotes statistical significance at the 1% level and * at the 5% level.

Internet Appendix

Table A1: Individual NCREIF Properties

We tabulate R^2 statistics of out-of-sample cash flow yield forecasts using two-quarter lagged appraisals (OOS $R^2_{appraisal}$) and the VAR model (OOS R^2_{VAR}) for all individual NCREIF properties, only NCREIF apartments, only NCREIF industrial properties, only NCREIF office properties, and only NCREIF retail properties over the sample period 1978:IV to 2013:II. The weight λ placed on the VAR forecast that minimizes the sum of squared combined out-of-sample cash flow yield forecasts is also tabulated along with the resultant combined forecast's R^2 statistics (OOS $R^2_{combined}$). We assess the uncertainty surrounding the λ estimates by block bootstrapping across all transactions during the sample period. We also provide the median percentage bias in an individual property's forecasted price using appraisals and the VAR model.

	all	apartments	industrial	office	retail
OOS $R^2_{appraisal}$	19.83%	18.96%	-4.66%	33.97%	36.50%
OOS R^2_{VAR}	28.53%	29.36%	24.08%	15.27%	23.73%
λ	0.5603**	0.5685**	0.6463**	0.4274**	0.4510**
OOS $R^2_{combined}$	48.61%	50.62%	43.20%	56.16%	65.51%
Appraisal price bias	2.50%	-3.06%	5.58%	4.36%	-4.59%
VAR price bias	0.97%	-8.49%	5.59%	2.50%	3.71%

** denotes statistical significance at the 1% level.

Table A2: Portfolios of NCREIF Properties

We form equal weighted portfolios (Panel A) and value weighted portfolios using appraised property values (Panel B) of out of sample cash flow yield forecasts. We tabulate resultant R^2 statistics using two-quarter lagged appraisals (OOS $R^2_{appraisal}$) and the VAR model (OOS R^2_{VAR}) for portfolios of all individual NREIF properties, only NCREIF apartments, only NCREIF industrial properties, only NCREIF office properties, and only NCREIF retail properties over the sample period 1978:IV to 2013:II. We also provide Diebold-Mariano robust test statistics of forecast encompassing ($DM_{encompassing}$) and equality of forecast accuracy ($DM_{accuracy}$) along with the time series estimates of the weight λ of the VAR forecast in the optimal combined forecast. We combine appraisals and VAR forecasts by using the single time series estimate of λ , this combined forecast's goodness of fit measured by OOS $R^2_{combined-1}$, and by using varying estimates of λ that minimize each month's sum of squared combined forecast errors, this combined forecast's goodness of fit measured by OOS $R^2_{combined-2}$.

Panel A: Equal Weighted					
	all	apartments	industrial	office	retail
OOS $R^2_{appraisal}$	70.48%	50.48%	33.43%	34.39%	50.71%
OOS R^2_{var}	50.42%	45.26%	30.00%	13.85%	39.25%
λ	0.3688	0.5475	0.5020	0.4844	0.3929
$DM_{encompassing}$	1.1990	2.0346*	2.3928**	2.0255*	1.9929*
$DM_{accuracy}$	-1.2845	0.4119	0.0162	-0.1342	-0.5276
OOS $R^2_{combined-1}$	77.27%	61.76%	52.38%	56.55%	58.77%
OOS $R^2_{combined-2}$	83.50%	72.90%	65.12%	76.58%	76.89%

Panel B: Value Weighted					
	all	apartments	industrial	office	retail
OOS $R^2_{appraisal}$	-18.66%	21.42%	-73.07%	-41.32%	7.54%
OOS R^2_{var}	51.99%	46.63%	37.00%	26.31%	35.18%
λ	0.8374	0.6399	0.8940	0.8391	0.6549
$DM_{encompassing}$	1.6233*	2.1521*	2.4365**	2.3161**	2.1277*
$DM_{accuracy}$	1.2541	1.1822	1.9663*	1.5049 ^o	0.8526
OOS $R^2_{combined-1}$	54.75%	58.32%	38.57%	28.89%	45.80%
OOS $R^2_{combined-2}$	43.69%	56.86%	21.63%	52.00%	52.13%

** denotes statistical significance at the 1% level, * at the 5% level, and ^o at the 10% level.